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## CONSIDERATIONS ABOUT THE INTERNODAL PERMEABILITY EVALUATION IN RESERVOIR SIMULATION

Jonas Cordazzo<sup>1</sup>, Clovis R. Maliska<sup>2</sup>, Regis K. Romeu<sup>3</sup>

<sup>1,2</sup> SINMEC – Computational Fluid Dynamics Laboratory – [www.sinmec.ufsc.br](http://www.sinmec.ufsc.br)  
Mechanical Engineering Department – Federal University of Santa Catarina – UFSC  
88040-900 – Florianópolis – SC – Brazil  
[jonas@sinmec.ufsc.br](mailto:jonas@sinmec.ufsc.br), [maliska@sinmec.ufsc.br](mailto:maliska@sinmec.ufsc.br)

<sup>3</sup> CENPES – Development and Research Centre – PETROBRAS – [www.petrobras.com.br](http://www.petrobras.com.br)  
Cidade Universitária – Avenida I Quadra 7 Ilha do Fundão  
21949-900 – Rio de Janeiro – RJ – Brazil  
[regis@cenpes.petrobras.com.br](mailto:regis@cenpes.petrobras.com.br)

**Resumo** – Este trabalho discute alguns problemas numéricos oriundos da utilização da média harmônica na avaliação da permeabilidade nas interfaces dos volumes de controle durante a simulação de reservatórios de petróleo e, depois, apresenta um método numérico que elimina esses problemas. O trabalho inicia realizando uma sucinta revisão bibliográfica e, na sequência, apresenta a forma como as permeabilidades são interpretadas pelos simuladores comerciais. Neste contexto, são comentadas as implicações da utilização das médias harmônicas nos resultados das simulações numéricas. Finalmente, o método de volumes finitos baseado em elementos (EbFVM) é analisado, destacando a questão da avaliação das permeabilidades. Este método, além de permitir a utilização simultânea de elementos triangulares e quadriláteros, é muito adequado para o tratamento das permeabilidades. As duas possibilidades de armazenamento das propriedades físicas deste método e suas implicações na avaliação da permeabilidade são apresentadas. O armazenamento da permeabilidade nos elementos, em vez da prática usual de armazenamento nos volumes de controle, elimina a necessidade da realização de médias controversas para o cálculo da permeabilidade nas interfaces dos volumes de controle.

Palavras-Chave: simulação numérica, reservatório de petróleo, permeabilidade, método de volumes finitos.

**Abstract** – This work reports some numerical problems caused by the using of harmonic average in the internodal permeability calculation during the petroleum reservoir simulation. This paper begins with a brief review, showing, in the sequence the approach used by commercial simulators in interpreting the permeability map. Then, the results of using harmonic averaging to determine the internodal permeability are presented by solving the 2D chess-board problem. Finally, the Element-based Finite Volume Method (EbFVM) is analyzed considering the permeability evaluation. It is demonstrated that this method, besides dealing with triangular and quadrilateral elements, can also deal with the permeability map without averaging. The two possibilities of storing the physical properties and their implications in the permeability evaluation are addressed. The permeability storage in the center of the elements, instead of in the center of control volumes, avoids the need of weighting the permeability values in the interfaces of the control volumes.

Keywords: numerical simulation, petroleum reservoir, permeability, Element based Finite Element Method.

## 1. Introduction

The geological modeling data used to depict the reservoir heterogeneity is often too much detailed to be directly handled by numerical simulators due to excessive number of grid points that would result. Therefore, an upscaling process is required, computing the effective properties on a coarse grid, trying to keep the original information obtained on the fine scale information. Among all properties to upscale, the absolute permeability has been the most studied. Several authors, Desbarats (1987), Romeu and Noetinger (1995), Ribeiro and Romeu (1997), among others, have analyzed the permeability upscaling.

The problem addressed here is related to the most widely used way to evaluate the internodal permeability in reservoir simulation. Usually, it is used an harmonic average of the two adjacent permeability block values, though some commercial simulators permit to use other values. This average can change the reservoir heterogeneity field interpreted by the simulator and introduces some error in the simulation results. For some cases, this error can be quite significant. These aspects are discussed in this paper.

Finally we present details about a numerical method that does not need to use any type of average in order to evaluate the internodal transmissibilities. Actually, in this method, the way to locate the variables and properties requires to calculate an average porosity, which is trivial, instead of an average permeability.

## 2. The Problem of Using the Internodal Permeabilities Calculated by Harmonic Average

The most popular numerical scheme for solving the flow equations in reservoir simulation applications is the central differencing scheme using five points in two dimensions and seven points in three dimensions (Aziz and Settari, 1979). In the discretization process, it is needed to evaluate the permeability in the interface between two gridcells. The most used weighting formula is the harmonic average (Pettersen, 1983):

$$k_{i+1/2} = \left( \frac{k_i^{-1} + k_{i+1}^{-1}}{2} \right)^{-1} \quad (1)$$

This type of average is physically justified for one-dimensional flow, but its extension to two and three dimensional situations is uncertain. Several authors have criticized the harmonic average used to evaluate the permeabilities (Toronyi and Farouq Ali, 1974; Appel, 1976; Pettersen, 1983), and the use of the arithmetic and geometric averages have been also proposed (Peaceman, 1977). In order to quantify the systematic bias in the numerical results due to the using of the harmonic average, Romeu and Noetinger (1995) developed an analytical approach in the form of a series expansion of the equivalent numerical conductivity in powers of the conductivity variance. The calculation confirmed the existence of a strong bias and of a very slow convergence in some cases. They proposed a simple method to correct it, which is well suited for upscaling.

Using laboratory experiments, Renard et al. (2000) investigated the impact of the numerical bias. They concluded that the differences in the estimated equivalent conductivity (or absolute permeability) using finite differences and finite differences is more important than the uncertainty in the equivalent conductivity resulting from the variability of the local scale conductivity. The impact of the bias is more important than the impact of the natural variability.

Ribeiro and Romeu (1997) examined the performance of three numerical methods – finite differences (FD), finite elements (FE), and mixed-hybrid finite elements (MHFD) – on the computation of the effective permeability in coarse and refined grids for some examples of heterogeneous media. They solved the problem depicted in Figure 1a, where the domain is subjected to no-flow boundary conditions along the perpendicular sides,  $P=1$  at inlet, and  $P=0$  at the outlet. The effective permeability in the  $x$  direction is given by  $QL/A$ , where  $Q$  is the total rate that enters or exits the system,  $A$  is the cross section area and  $L$  is the length shown in the Figure 1a. The permeability map is shown also in the Figure 1a.

The reference solution ( $k_{eff} = 37.71$ ) was calculated refining the original 4x4 grid up to 700x700. Ribeiro and Romeu (1997) concluded that the FE overestimates the solution for  $k_{eff}$ , and both FD and MHFE underestimate the solution for  $k_{eff}$ . However, all of these methods converge to the reference solution when the grid is sufficiently refined. Figure 1b shows how the three methods converge when the refinement ratio,  $r$ , increases. We can note that, for this example, the arithmetic average of the FD and FE results is a quite accurate estimate, even for coarse grids. However, the authors emphasized that there are other cases where this average does not work.

Therefore, due to the harmonic average done in commercial simulators in order to estimate the internodal permeabilities, some simulation results, sometimes, can result in an erroneous heterogeneous map. Figure 2a, for example, shows a heterogeneous media where the permeability values of the grid-blocks are set to 1000 and 1, alternately. The simulator interprets this permeability map, because of harmonic average, like the one shown in Figure 2b. One can see that the new permeability map is now homogeneous, and the all grid-block permeabilities are set to the value obtained by harmonic average between the values 1000 and 1, that is 1.98. Obviously this new map no longer

represents the features before the upscaling process, and any simulations results, based on this new map, may be completely wrong.

One possible solution for this problem is to refine the grid. Using a refinement ratio  $r=2$  in the grid shown in Figure 2a, we have the grid shown in Figure 2c. In this case, the simulator interprets this permeability map as the one presented in Figure 2d, where the permeability values are set to 1.98 only in interfaces of the regions that have different permeabilities (1 or 1000). The gray region, with  $k=1.98$ , in Figure 2d decreases as the refinement ratio increases. We can note that only with a grid refinement procedure the simulator can interpret the grid as provided by upscaling process. Although one needs to refine the grid in order to reduce the truncation error of the numerical approximation of the partial differential equation, the refinement required for reducing the error caused by the harmonic average is more severe. Therefore, extra computational effort is demanded for reducing the errors caused by using averaging in the properties.

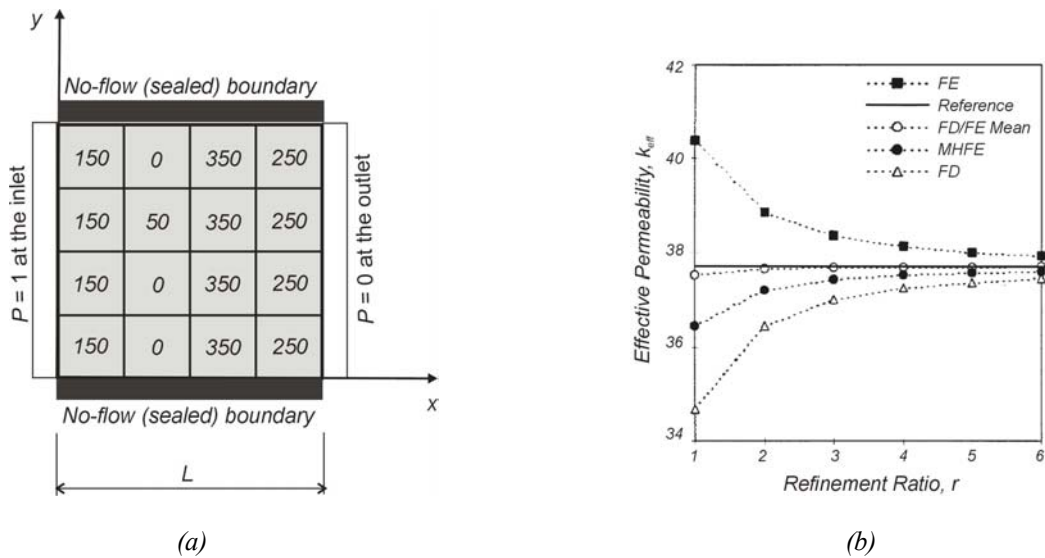


Figure 1. (a) Boundary conditions and permeability map used to compute the effective permeability, (b) Values of  $k_{eff}$  computed by three numerical methods for different levels of grid refinement (Ribeiro and Romeu, 1997)

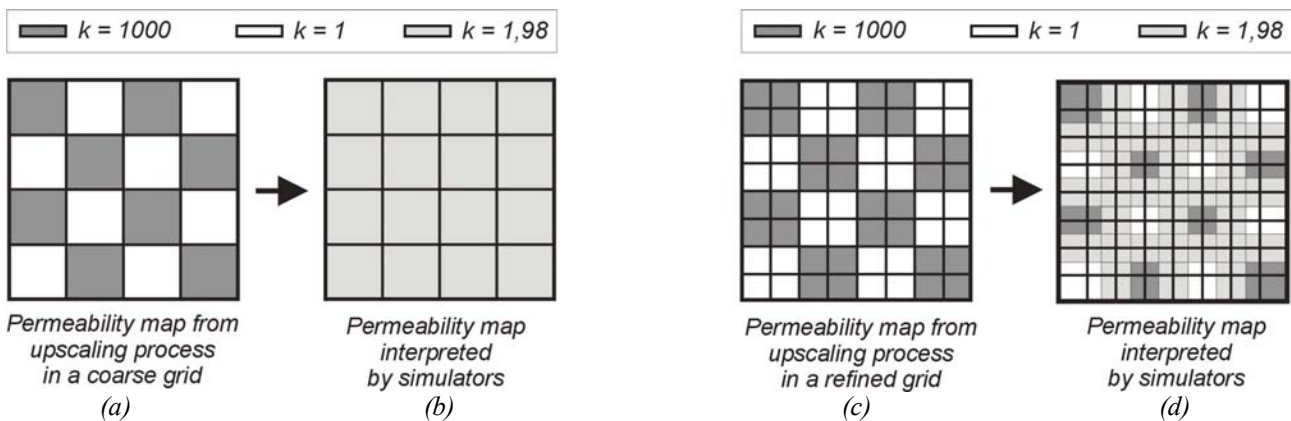


Figure 2. (a) and (c) Permeability map from upscaling process in a coarse and refined grid, respectively; (b) and (d) permeability map interpreted by simulators in a coarse and refined grid, respectively.

### 3. The EbFVM Method and its Permeability Approach

The difficulty encountered when some averaging procedure is done is related to the relative storing of variables and properties in the computational grid. Denoting the element as the entity which is formed by the grid-lines and the control volumes the entity in which the balances of the properties are made, the need for some type of average to calculate the internodal permeability will depend on the relative geometrical position of control volumes and elements.

The calculation of any property or geometrical information at one integration point should depend only on values of the variable and geometrical data stored at the nodes that defines the element (Maliska, 2002). However, most of the numerical schemes have used the element coincident with the control volume during the integration of the partial differential equation. In this construction, called cell centered, a variable to be calculated at the integration points will need values from different elements. This is the reason why the averaging is required, leading to the difficulties above discussed.

The cell vertex construction is the other approach for constructing the control volumes. In this construction, the control volumes are created joining the center of the elements to its medians. In this case, all fluxes at one specified integration point can be calculated using data from the element where the integration point lies. This feature makes this approach very adequate for reservoir simulation, because the physical properties can be stored in the elements instead of the control volumes. So, as the integration points are inside the elements, there is no need to do any type of average to calculate the permeability in these points, since the elements are homogeneous, i.e. each element has only one value of permeability. This method was presented by Cordazzo (2002) and it is briefly considered in this section, with the focus in the aspects related to the permeability evaluation.

Following, therefore, it is presented the basic ideas of a numerical reservoir simulator under development that presents an adequate location of permeabilities. It employs the ideas of Raw (1985) when developing the FIELD method for solution of the Navier-Stokes equations. It belongs to the class of the Element-based Finite Volume Methods (EbFVM) with new features for mobilities evaluation, relative and absolute permeabilities and local refinement near wells and/or faults. EbFVM is a better denomination for the method (Maliska 2003), also known as Control Volume Finite Element Method (CVFEM) since it is a finite volume methodology that borrows from the finite element technique the concept of elements. The usual denomination, CVFEM, would suggest a finite element formulation that obeys the conservation principles at discrete level. The motivation for using this methodology is its flexibility, generality, and clean computational implementation.

In a finite volume methodology the domain is covered by non-overlapping control volumes where the balances are done, as shown in Figure 3 where an element is also defined. As already stated, in the cell vertex construction, the control volumes are created joining the center of the elements to its medians. The resulting control volume is formed by portions (sub-control volumes) of neighboring elements. In this case, all fluxes at one specified integration point can be calculated using data from the element where the integration point lies. This allows the creation of the approximate equation for the unknown variable located at the center of the control volume, which is formed by sub control volumes of the neighboring elements.

For quadrilateral elements, even in Cartesian grids, this method results in a nine-point scheme for the pressure. The use of nine-point scheme has been considered a desirable feature in numerical methods in order to reduce grid orientation effect (Yanosik and McCracken, 1979; Ko and Au, 1979; Shiralkar and Stephenson, 1987).

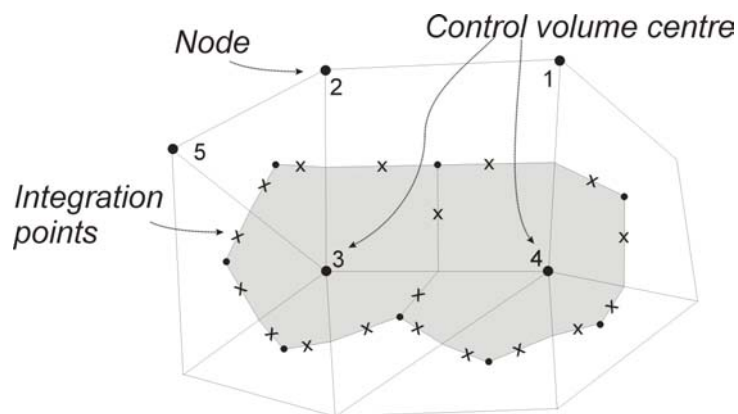


Figure 3. Element, volume and sub-volume with integration points in a corner point grid (Maliska, 2002)

It is important to mention that in this method, the physical properties, like absolute permeability and porosity, are stored in the centre of the elements, instead of the other models even other CVFE, that store the physical properties in the control volumes centre. Even though using this scheme of properties storage, the properties calculated in the model, like pressure and saturation, are still stored in the grid-nodes. Figure 4 compares the usual scheme of the commercial simulators with the one proposed in Cordazzo (2002).

The main advantage of the Element Based Finite Volume Method is the need of dealing with only one grid (the elements), which is built using triangles and/or quadrilaterals. In the center of the elements it is stored the physical properties like absolute permeabilities, porosity etc.. This results in an easy procedure to build grids that represents the heterogeneities with more fidelity. From a practical point of view of the simulator users, this method demands to “see” only a grid, which is built only by elements, instead of other methods where there is the need of dealing also with control volumes. Therefore, in the EbFV method the control volumes are heterogeneous, instead of the elements, as in

other methods. Therefore, as the integration points are inside the elements, there is no need to make any average to calculate the permeabilities at the control volume interfaces. Therefore, the problems related to the internodal permeability evaluation shown in the previous section do not exist in the proposed EbFV method.

In the homogeneous elements approach, the transient term of the partial differential equation should be rewritten. Thus, some type of average should be made in order to estimate a control volume representative porosity. Cordazzo (2002) proposed a weighted average porosity as a function of the volume of each sub-control volume. Therefore, the transient term integrated over the sub-control volume ( $scv$ ) results in

$$\int_{scv} \frac{\partial(\phi S / B)}{\partial t} dV = V \frac{(\bar{\phi} S / B)^{n+1} - (\bar{\phi} S / B)^n}{\Delta t} \quad (2)$$

where  $S$  is the saturation,  $B$  is the formation volume factor,  $V$  is the volume,  $t$  is the time, the superscript  $n$  is the temporal level, and  $\bar{\phi}$  is the average porosity given by

$$\bar{\phi} = \frac{1}{V} \sum_{k=1}^{N_{scv}} V_k \phi_k \quad (3)$$

where  $N_{scv}$  is the number of sub-control volumes that compose the control volume.

There are at least two reasons why we can say that taking a mean value of porosity is not so troublesome than taking a mean value of permeability. First, often the range of variation of permeabilities values is greater than the variation of porosity values in a field. Second, the permeability is a term appearing in Darcy Law, while porosity is not. We should remember that the Darcy law is the momentum equation for a porous media. These reasons justify the using a numerical method that uses a average porosity value instead of a average permeability value.

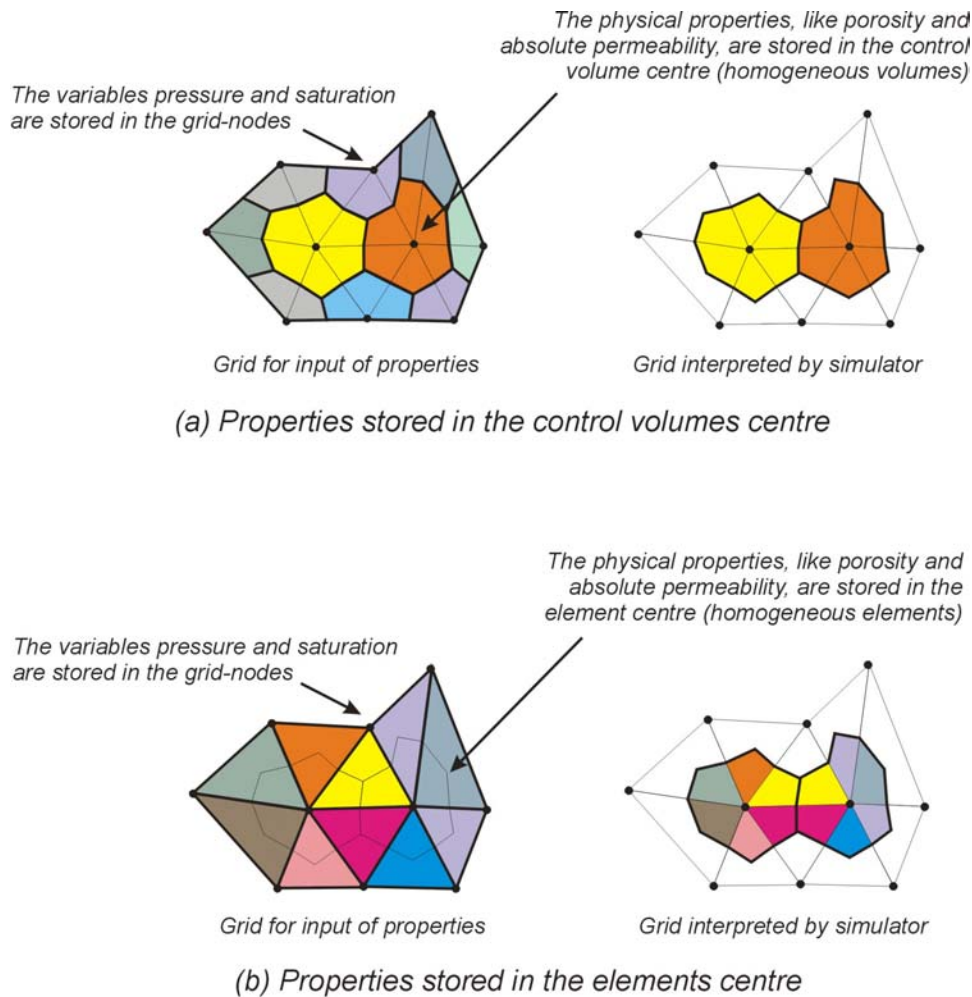


Figure 4. Different ways to store the physical properties: a) in the control volume centre, and b) in the elements centre (proposed by Cordazzo, 2002)

## 4. Conclusions

This paper briefly discussed the key problem of averaging the permeability in petroleum reservoir simulation. It was seen that a severe grid refinement is necessary if averaging procedure are employed in order to well represent the reservoir. It was also made clear that the need for the averaging results from the relative storing of variables and properties in the computational grid. Usually, properties and variables are stored in the center of the control volume, since most methods do not use the concept of elements. After recognizing the difficulties, it is related a proposed element-based finite volume method where the concepts of elements and control volumes are used. It was demonstrated that storing the properties in the center of the elements and variables in the center of control volumes there is no need for any permeability averaging.

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