RESERVOIR AND WELL COUPLING SIMULATION WITH DISTRIBUTED ALGORITHM

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ABSTRACT

A major goal when using a horizontal well is to increase the productivity rate when compared to the vertical layout, due to a larger area of the reservoir exposed to the well. The coupled solution into the reservoir and through the well requires finding the answer to the Darcy equation in the reservoir and the Navier-Stokes equations in the well. Additionally, several types of completion, which introduce new flow resistances, such as completions with inflow control device (ICD), stinger completion, packed off, gravel pack, and slotted liner are considered. All these details are not available with the current reservoir simulators, since the level of local detail would require a very large computational time and a complex model. In this work, the coupling between the reservoir and the well is accomplished through a distributed system, where the flow, at various types of completion, is handled through correlations obtained analytically or provided by completion equipment manufacturers. The proposed procedure consists of a fast tool, accurate and easy to use for a petroleum engineer. The results of this work show that the distributed system allows a detailed view of the flow into the well and its completion, quantifying the flow rate and pressure over the annular and liner for different types of completion, adding more physics into these simulations.

KEYWORDS

reservoir well coupling; distributed algorithm; well completion; unconventional completion; ICD; horizontal wells

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1. INTRODUCTION

The domain discretization in the universe of reservoir simulators commonly employs elements with large dimensions. Due to this kind of approach, it is usual to disregard the geometric effects of each well on the drainage, often using simplified assumptions to estimate the fluid dynamic effects from wells in the reservoir. In horizontal wells, it is evident that the well reservoir coupling becomes more delicate, since the well has a much larger extent in contact with the reservoir, and the forms of completion are not easy to compute, especially when using a flow equalization system along the well.

The first to develop a semi-analytical model for reservoir well coupling was **Dikken (1990)**. With this model, he proved the importance of horizontal well friction to show that the inflow per unit length in the well decays with the increase in well length, and that most of the flow in the reservoir moves towards the well heel. This effect causes an earlier breakthrough of water or gas in the well.

Other authors (**Ozkan et al., 1999; Dickstein et al., 1997**) also reported that the hypothesis of infinite conductivity in the well can be applied restrictedly to systems where the pressure gradient in the well is negligible compared to the reservoir pressure loss (drawdown). Later, other researchers gave a numerical treatment to reservoir and well coupling, discretizing the well in various segments (**Holmes et al., 1998; Kabir & Sanchez, 2009; Brekke & Thompson, 2009**).

In this work, a computational simulator using an algorithm of distributed system is presented, it solves the momentum balance equations in the well (along the liner and annulus), the flow equations for different types of completions (between the annulus and liner), and also the flow between the reservoir and the well in the region of influence of the well, with the objective of determining the pressures and flows throughout the entire domain. The model has the ability to account the recent advances in the well completion schemes such completion with uneven perforated liner, completions with multiple ICD's, and stinger completion. This simulator considers single-phase or two-phase flows (oil and gas) with homogeneous model.

2. METHODS

The distributed approach is to discretize the well and the near well region in several segments connected by nodes, representing all components (liner, annulus, and completion) and also the connection with reservoir nodes, as illustrated in Figure 1.

In the structure of the distributed system, the boundary conditions of the problem are imposed by external nodes. At the Nodes of the reservoir are provided pressures and reservoir saturations, which can be obtained from a reservoir simulator. The last node of the liner (bottomhole) is provided the bottomhole pressure.

Figure 1. Distributed system structure.

There are 9 unknowns associated with each segment: pressure at liner node, pressure at annulus node, inlet flow rate, flow rate in liner segment, flow rate in annulus segment, flow rate in the segment between the annulus and liner node, oil phase fraction at liner segment, oil phase fraction in annulus segment, and oil phase fraction in the segment between the annulus and liner node.

The equations are obtained from the material balance at each node of the system (considering the compressibility and the mass transfer between phases, according to the black-oil model) and momentum balance in each system connection. This set of nonlinear equations is solved simultaneously through the iterative Newton-Raphson method.

2.1 Mass balance equation for oil and gas

The mass balance at each node of the distributed system, without mass accumulation in this system, for the two-phase flow oil and gas are:

$$\sum \dot{m}_o = \sum \frac{q\alpha}{B_o} = 0 \quad (1)$$

$$\sum \dot{m}_g = \sum \left(\frac{q(1-\alpha)}{B_g} + \frac{R_s q \alpha}{B_o} \right) = 0 \quad (2)$$

At these equations, the flow entering the nodes is positive and the one leaving the nodes is negative, $\dot{m}_o$ and $\dot{m}_g$ are the oil and gas mass flow rates, B_o and B_g are the formation volume factors of oil and gas, α is the volume fraction of oil, R_s is the solubility ratio of gas in oil, and q is the volumetric flow rate (oil and gas) entering or leaving the node.

2.2 Coupling equation

The equation to be solved in the connection between the reservoir nodes and the annulus nodes is the analytical solution represented by the Darcy equation for a porous media with cylindrical geometry and radial multiphase flow. Considering the connection between nodes A and B of Figure 1:

$$q_{A,B} = \frac{2\pi K L}{\left[\ln\left(\frac{r_d}{r_w}\right) + s \right]} \left(\frac{k_{ro}}{\mu_o} + \frac{k_{rg}}{\mu_g} \right) (p_A - p_B) \quad (3)$$

Where r_d is the well drainage radius, r_w is the well radius, s is the reservoir damage factor (skin) on the segment, L is the length of the well segment, $q_{A,B}$ is the volumetric flow entering the well at that segment, and K is the absolute permeability of the reservoir in the segment. The value of K may vary from one segment to another, enabling to take into account heterogeneity of the reservoir, μ_o is the oil viscosity, μ_g is the gas viscosity, p_A is the pressure in the reservoir node (boundary condition), p_B is the pressure in the annulus and k_{ro} and k_{rg} are the relative permeability to oil and gas, respectively.

2.3 Momentum balance equation in the well

At the connections between two annulus nodes or between two liners nodes, the momentum balance equation is solved, taking into account the effects of friction, gravity, and acceleration.

$$\left(\frac{dP}{dL} \right) = - \frac{\rho_m v_m^2 f}{2d} - \rho_m g \sin \theta - \frac{\dot{m}}{A} \frac{dv_m}{dL} \quad (4)$$

The term of the pressure gradient due to gravity is disregarded because the experiment used horizontal wells. The term of the pressure gradient due to the acceleration, caused by the increase in flow velocity with the radial fluid inlet, is also disregarded since the momentum transfer in the radial direction is significant only at the beginning of the well, near the beginning of the well's toe. As the flow moves toward the heel of the well, it becomes negligible when compared with the momentum in the axial flow direction. Therefore, equation (4) becomes:

$$\frac{dp}{dL} = \frac{\rho_m v_m^2 f}{2d} = \frac{\rho_m q_m^2 f}{2dA^2} \quad (5)$$

Where, ρ_m is the mixture density, q_m is the volumetric flow (annulus or liner), f is the friction factor (Darcy-Weisbach), d is the diameter (annulus or liner), A is the cross section of the flow (annulus or liner). The friction factor is determined by (Haaland, 1983) an explicit correlation with excellent agreement with the implicit equation of Colebrook, according to (Dejan, 2011).

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10} \left(\left(\frac{\varepsilon}{3.7D} \right)^{1.11} + \left(\frac{6.9}{Re} \right) \right) \quad (6)$$

In this equation, ε is the absolute roughness (annulus or liner) and Re is the Reynolds number of the flow (annulus or liner). Applying equation (5) according to Figure 1, for connection between nodes B and E of annular considering the hydraulic diameter of the annulus and also to the connection between nodes C and F from liner:

$$p_B - p_E = \frac{\rho_m q_{B,E}^2 f L}{2 d_h A_a^2} \quad (7)$$

$$p_C - p_F = \frac{\rho_m q_{C,F}^2 f L}{2 d A_l^2} \quad (8)$$

The momentum balance equation in the connections between the annulus and liner nodes depends on the completion scheme used and can be represented by the equation of pressure loss in holes. According to **Kaiser et al. (2000)**, this pressure loss is caused mainly by the convergence of flow around the holes than by friction on the walls of the holes. Considering the connection between nodes B and C of Figure 1:

$$p_B - p_C = \frac{q_{B,C}^2 \rho_m}{2 C_d^2 n^2 A_{hole}^2} \quad (9)$$

Where p_B and p_C are, respectively, the pressures at the nodes of the annulus and liner; $q_{B,C}$ is the volumetric flow rate (oil and gas) in the connection; ρ_m is the density of the mixture in the connection; C_d is the discharge coefficient of the holes; n is the number of holes in that segment of the well; and A_{hole} is the area of each hole. From the results of **Miller et al. (2009)** and **Hollingshead (2011)**, the discharge coefficient depends on the Reynolds number of the holes.

2.4 Split equation

The above types of models give 8 equations associated with each segment (2 mass balance equations for the annulus node, 2 mass balance equations for the liner node, 1 inflow equation, and 3 momentum balance equations). In order to solve 9 unknowns, the governing equation system needs another equation to make the solver work, which is provided through "split equations."

For annulus nodes where fluids exit, splitting in two different directions, the authors assume that individual oil phase fraction in the directions are equal. For instance, according to Figure 1, for connection between nodes B and E and for to the

connection between nodes B and C:

$$\alpha_{B,E} = \alpha_{B,C} \quad (10)$$

2.5 Solution of the distributed system

Mass balance is applied to the nodes and the momentum balance is applied to the segments, resulting in a nonlinear system which is solved by the Newton-Raphson method. The system of nonlinear equations can be written as follows:

$$\begin{aligned} F_1(x_1, x_2, \dots, x_n) &= 0 \\ F_2(x_1, x_2, \dots, x_n) &= 0 \\ &\vdots \\ F_n(x_1, x_2, \dots, x_n) &= 0 \end{aligned} \quad (11)$$

The Jacobian matrix J , the $\vec{F}$ vector, and the $\Delta\vec{x}$ vector are used to solve simultaneously the nine unknowns of nonlinear system:

$$J \cdot \Delta\vec{x} = -\vec{F} \quad (12)$$

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \dots & \frac{\partial F_2}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \dots & \frac{\partial F_n}{\partial x_n} \end{pmatrix} \quad (13)$$

$$\Delta\vec{x} = \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{pmatrix} \quad (14)$$

$$\vec{F} = \begin{pmatrix} F_1(x_1, \dots, x_n) \\ F_2(x_1, \dots, x_n) \\ \vdots \\ F_n(x_1, \dots, x_n) \end{pmatrix} \quad (15)$$

The new values of the unknowns $\vec{x}$ are calculated by:

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \Delta\vec{x}^{(k)} \quad (16)$$

Table 1. Parameters used in the simulations.

PARAMETER	Case 1	Case 2	Case 3
Completion type	ICD	Uneven perforated liner	stinger
Well length (m)	1000	1000	1000
Number of sections	100	100	100
Reservoir pressure (bar)	380	380	380
Pressure at heel (bar)	375	375	375
Bubble pressure (bar)	290	290	290
Drainage radius of the well (m)	20	20	20
Well diameter (in)	6.5	6.5	6.5
Liner diameter (in)	5.0	5.0	5.0
Absolute permeability of the reservoir(mD)	800	800	800
Skin factor	1.0	1.0	1.0
Oil saturation in the reservoir	1.0	1.0	1.0
Oil viscosity (cP)	1.25	1.25	1.25
Oil density (Kg/m ³)	780	780	780
Oil formation volume factor (m ³ / m ³ std)	1.34	1.34	1.34
Absolute roughness of liner (m)	6e-5	6e-5	6e-5
Absolute roughness of formation (m)	6e-3	6e-3	6e-3
Open area in the liner (%)	variable	variable	6

The iterative scheme used to solve the distributed system is detailed below:

1. Discretize the computational domain formed by the well (annulus, liner, and completion) and the one near well region into segments connected by nodes, imposing the boundary conditions;
2. Estimate the initial values for the unknowns (p , q and α) along the domain;
3. Calculate the fluid properties and the coefficients of the Jacobian matrix;
4. Solve the linear system $J \cdot \Delta \vec{x} = -\vec{F}$ to determine the new values of unknowns and compare them with the values of the previous iteration;
5. Repeat steps 3 and 4 until the convergence of the unknown's values.

3. RESULTS AND DISCUSSIONS

From the results presented, one can observe the capability of the distributed system developed here to model the recent advances in the well completion schemes, simulating three types of completion: completion with multiple inflow control devices (ICD), uneven perforated liner and

stinger completion. The input parameters for the simulations in single phase flow are shown in Table 1.

3.1 Completion with multiple inflow control devices (case 1)

The inflow control device (ICD) acts as a flow restrictor between the annulus and liner, which is part of the well completion, and it works as a system of equalization of flow along the well. In this simulation, it was considered a liner with uniform drilling (6% open area to flow) in the first 500 meters of the well and in the next 500 meters a liner without holes with 2 ICD's installed, both with 0.2% open area to flow, one at 250 meters of the heel and the other at the heel. A scheme of this type of completion is shown in Figure 2.

At Figure 3, one can see that throughout the liner without holes (last 500 meters), the pressure and flowrate rise in the annulus. Arriving at ICD's, the accumulated fluid in the annulus flows to the liner, relieving its pressure. This causes the pressure difference between the reservoir and the annulus (drawdown) tends to be uniform along the well due to the presence of the two ICD's, resulting in a more uniform drainage of the reservoir,

Figure 2. Types of completion.

Figure 3. Pressure and flowrate distribution along the well (Case 1).

delaying the occurrence of the breakthrough. It is also observed in Figure 3 that with this type of completion, the produced flow was $11500 \text{ m}^3/\text{d}$ for the parameters in Table 1.

3.2 Completion with uneven perforated liner (case 2)

This type of completion consists in the use of a liner with variable drilling. The open area to the flow in the liner varies over the length of the well, having a maximum value at the toe, decreasing as it approaches the heel. The objective is to compensate the pressure drop within the well making the pressure more even over the annulus and providing a more uniform drawdown, delaying the breakthrough. A full description of this type of

completion can be obtained in [Fernandes et al. \(2006\)](#). Figure 2 illustrates this completion.

A 1000-meter length well, with 6.5" diameter and a 5" liner is considered. Its liner has 10 different perforations along its length. The first 100 meters from the toe have 5% of open flow area, this open area decreases every 100 meters, until reaching the last 100 meters near the heel with an open flow area of 0.003%.

At Figure 4, the pressure outline in the well shows that the annulus pressure tends to be uniform due to the liner with variable drilling. Therefore, the pressure difference between the annulus and the reservoir also tends to be uniform, resulting in a more uniform drainage, avoiding the occurrence of gas or water cone, delaying the

Figure 4. Pressure and flowrate distribution along the well (Case 2).

Figure 5. Pressure and flowrate distribution along the well (Case 3).

breakthrough. Due to pressure uniformity in the annulus, as one can see in Figure 4, the annulus flow is lower, reaching a maximum value of 1000 m³/d. For the parameters in Table 1, with this type of completion, the produced flow was 9400 m³/d.

3.3 Stinger completion (case 3)

This type of completion consists in dividing the liner into two parts, one part is completed with a maximum open area to flow (6%), and the other is completed with stinger (blank pipe). The stinger completion is introduced to balance the unequal pressure drawdown along the well from toe to the heel, uniformizing the drainage.

This simulation used a liner with uniform drilling in the first 500 meters of the well and in the next 500 meters a liner without holes. The physical

structure and fluid movement is shown in Figure 2.

In Figure 5, it appears that the annular pressure profile tends to be more uniform than the pressure profile in the liner due to the stinger presence. One can observe that the drawdown is smaller in the region of the stinger, causing this region to have smaller reservoir drainage. The flow in the liner increases over the open liner, until it reaches the junction point with the stinger. At this point, all the annular flow is transferred to the liner and its value remains constant until the heel of the well.

The annular flow gradually increases from the toe and from the heel towards the junction point, where all this flow is transferred to the liner. With this type of completion, for the input data provided by Table 1, the production was 5600 m³/d.

4. CONCLUSIONS

In solving complex problems in engineering, such as the well reservoir coupling, it is extremely advantageous to have fast and simple solutions, which provide good accuracy in the results. The computer simulation method of the well reservoir coupling using a distributed approach, demonstrated here, follows this line of thinking.

The robustness and consistency of this iterative technique, illustrated by simulations, consider three different completions (ICD's, uneven perforated liner, and Stinger). The results obtained produced a very detailed analysis of completions studied, infeasible to obtain with current reservoir simulators, showing that the distributed system is a powerful, easy to use, and low computational cost simulation in the well reservoir coupling simulation with a detailed look at the well, especially when equipped with special systems of completion aimed at equalizing flow, which can be used both to design as to predict problems in wells.

Comparing the three types of completion, it appears that according with the flow curves, the completion with multiple inflow control devices (ICD's) had the highest production (11500 m³/d), followed by completion with uneven perforated liner (9400 m³/d), and lastly the stinger completion with 5600 m³/d.

The difference between the productions of the completions occurs because the average drawdown (the pressure difference between the reservoir and the annulus) is highest in completion with ICD's and lowest in the stinger completion, as show in the pressure curves.

One can also observe that, in the flow curves, the reservoir drainage along the well is more uniform in the completion with uneven perforated liner, where the inflow is quite linear, followed by completion with ICD's, and finally the Stinger completion. In the Stinger completion can be verified that in the last 300 meters near the heel, the reservoir drainage is less than the rest of well, indicating a non-uniform drainage in this region.

According with the results, for the scenery presented, the completion with inflow control devices had the better productivity, and the completion with uneven perforated liner had the more uniform flow distribution along the

horizontal well and, therefore, more uniform drainage of the reservoir.

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