NUMERICAL SIMULATION OF NATURALLY FRACTURED SHALE GAS RESERVOIRS APPLYING THE LAYER MODEL

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ABSTRACT

Typically, hydrocarbon reservoirs have heterogeneities that influence pressure variation in producing wells. In the case of natural fractures, the layer model has been widely applied to obtain analytical and numerical solutions. In this model, we superimpose the porous media layers representing the porous matrix and fractures. In this work, the layer model is used in the numerical simulation, using cylindrical geometry and finite differences, to determine the pressure in natural gas producing wells. We consider shale gas reservoirs, and obtain the numerical results investigating the effects resulting from the presence of fractures and non-Darcy flow due to slip and adsorption phenomena. This work studies grid refinement and performs numerical verification, as well as a sensitivity analysis varying the main physical parameters that directly influence the reservoir and well pressures, such as permeability, porosity, and thickness of the fracture, reservoir temperature, and the Langmuir pressure and volume. As expected, from specialized and diagnostic plots, we can visualize how fractures favor the flow while capturing typical flow regimes of fractured reservoirs described in the literature.

KEYWORDS

adsorption; layered model; naturally fractured reservoir; shale gas; slip flow

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1. INTRODUCTION

Nature is strongly affected by its interaction with human beings. The uses of oil can be traced to different civilizations, dating from times before the birth of Christ. Its applications on a commercial scale, however, only began in the second half of the nineteenth century, being the most prominent energy source in the global energy matrix in the twentieth century, with intensification after World War II.

The use of natural gas (fossil origin) has also appeared in historical records. Documents from China point out to its application in warming brines in the process of obtaining salt even before Christ. However, for commercial use, natural gas has not been around for a long time due to the difficulty of extraction, transport, and storage when compared to oil. Thus, technological advances coupled with an increased demand for energy (Guo & Ghalambor, 2014), gave natural gas an important place in the global energy matrix throughout the twentieth century. Some energy experts make the claim that the 21st century may be the century of natural gas. It competes with oil in terms of breadth of applications and offers a lower pollution potential. Due to the recent development in the oil and gas industry, unconventional natural gas reservoirs, which require advanced techniques to make their production feasible, have also contributed to increased gas production. This paper examines one of these types of reservoirs, precisely, shale gas (Wang et al., 2017).

1.1 Shale gas

Shale is a fine-grained rock with a varied mineralogical formation (Jia et al., 2018), it is formed by minerals such as carbonates, quartz, and feldspar (Jacomo, 2014). Shale-like formations have a very low absolute permeability, between 10^{-6} and 10^{-9} Darcy (Ali, 2012), and porosity (below 0.15). The higher its permeability is, the smaller the pressure variation in the reservoir for production at a fixed flow rate is. Nevertheless, the higher the porosity, the more fluid can be produced, favoring hydrocarbon recovery.

Despite the low permeability and porosity values of these reservoirs, gas is a fluid that has low viscosity. This characteristic contributes to having a fixed rate production with less pressure variation.

Still, the extent of these reservoirs offset the low porosity. The development and application of horizontal wells and hydraulic fracturing techniques has led to an increase in production over the past two decades (Wang, 2013).

1.2 Shale gas production features

The characteristics of the rock-fluid system may also contribute to the production in shale gas reservoirs (Rosa et al., 2006; Li et al., 2016). Horizontal wells can reach kilometers in length, and their main objective is to favor, or even make viable, the production of reservoirs in which vertical wells would not be an economically viable alternative. They allow a larger contact surface of the porous medium with the well surface open to the flow, so that it can produce at a constant rate with less pressure variation when compared to vertical wells (Chung et al., 2017). In the case of hydraulic fracturing, fluid is injected at high pressure into reservoirs to break the rock and facilitate the flow (Chung et al., 2017). Fluids with proppants are injected to prevent fractures from closing. However, fracturing operations may lead to groundwater contamination and small earthquakes. Figure 1 schematically illustrates a hydraulically fractured horizontal well.

The use of hydraulically fractured horizontal wells in shale gas production has increased considerably in the last twenty years, mainly in the USA. In fact, with oil prices falling over the past five years, the shale gas industry has suffered a setback, with many companies ending their activities because they are no longer competitive. However, the current scenario showed the robustness of the remaining companies. Besides that, we have identified the presence of a consolidated market with the new prices of oil and gas. According to Hasan et al. (2013), two-thirds of

Figure 1. Hydraulically fractured well.

the world's hydrocarbon reserves are unconventional, and this fact is directly related to the growing importance of shale gas in the global energy matrix. In a scenario of technical challenges and market rebalancing, the shale gas industry is generating major technological breakthroughs (Gomes, 2011).

As already mentioned, other factors linked to the nature of rock and fluid help the production in shale gas reservoirs. Literature highlights the slip and adsorption effects of the gas (Li et al., 2016; Jiang & Younis, 2015). In the case of slipping, the gas that is in direct contact with the solid surface can slip on the surface. In adsorption, the rock surface traps a portion of the gas. As the pressure drops, the rock surface may release the gas. Another factor related to the nature of the rock that can help for gas production is the presence of natural fractures. A natural fracture is a planar discontinuity present in the rock (Figure 2), which originated from deformation or a diagenesis (Tiab & Donaldson, 2004). Natural fractures vary greatly depending on their distribution and size, and may lead to the formation of channels, fracture networks, or even vugs.

In the case of carbonates, high permeability channels and vugs arise from the dissolution of the rocks. According to de Souza and Amaral Souto (2014), the Naturally Fractured Reservoirs (NFRs) flow modeling considers the existence of two regions, fracture and matrix. The matrix contains the fractures. In general, fractures have higher permeability and porosity values than those of the matrix. Therefore, the occurrence of natural fractures in shale gas reservoirs also facilitates production (Heinemann & Mittermeir, 2014). Nelson (2001) presents a classification of naturally fractured media according to the effects they have

on the reservoir:

1. Fractures provide both porosity and permeability for productivity;
2. Reservoirs have low porosity and permeability in the matrix, and fractures provide the essential permeability for productivity;
3. Reservoirs have high porosity and can produce without fractures, and fractures provided added permeability;
4. Reservoirs have high porosity and permeability, and fractures add no significant additional porosity and permeability.

Over the years, several authors have studied the flow in naturally fractured reservoirs, such as the classic works of Warren and Root (1963) and Kazemi (1969) that use the layer model, among others (Thomas et al., 1983; Ishimoto, 1988; Tavares, 2003; de Souza et al., 2010; Debossam et al., 2019). In the layer model, layers of porous matrix and fracture are interposed in the representation of the reservoir. The influence of fractures becomes even more relevant in the case of shale gas reservoirs (Jiang & Younis, 2015; Li et al., 2017), due to the low reservoir permeability and porosity values. Over time, the main characteristics that influence production have been incorporated into reservoir numerical simulation models and well pressure analysis. The numerical test analysis is an intersection of these two areas of reservoir engineering.

1.3 Numerical test analysis

The main objective of reservoir engineering is to maximize the hydrocarbon recovery factor, which comprises several areas such as reservoir modeling, material balance, production decline analysis, history adjustment, recovery methods, reservoir numerical simulation, and analysis of well pressure tests (Rosa et al., 2006). This paper deals with the confluence of pressure test analysis with reservoir numerical simulation. Well pressure test analysis (Bourdet, 2002) aims to characterize the well-reservoir system via the solution of inverse problems, considering models for well-reservoir flow and the use of data from well pressure tests. Thus, it is possible to obtain, for example, reservoir pressure, formation permeability, and reserve estimates, which are accurate information on the

Figure 2. Naturally fractured reservoir.

characteristics of the production system. This way, it is possible to adopt a better strategy for production. In a constant flow rate pressure test, for example, the pressure in the well drops over time, and a pressure recorder located in the well collects data. Computing in the area of pressure test analysis is widely used in the oil industry (Kappa Engineering, 2018; Fekete Associates, 2019; Offshore Energy, 2019).

According to Ertekin et al. (2001), in the numerical simulation of reservoirs, the partial differential equations that govern the flow undergo a discretization process, which is their conversion into algebraic equations that lead to a system of equations. When such a system is solved, for example, for a single-phase flow, the pressure field of the reservoir is obtained for given initial and boundary conditions and for a given operating scenario. It is also possible to obtain, via numerical simulation, estimates of pressure in the wells (de Souza, 2013).

The main objective of this work is to perform numerical simulation of pressure tests in shale gas reservoirs, such as test planning, including the effects of slipping and adsorption, incorporating the presence of natural fractures through the application of the layer model and using cylindrical coordinates for the capture of the physical effects close to the well. Another possibility is the use of the simulator in conjunction with some test techniques to solve an inverse problem, aiming to characterize the well-reservoir system. Therefore, the simulations performed in this work are in the context of well pressure tests. It has been widely used in this model in the study of fractured media for problems involving oil, gas, or multiphase flows.

2. MATERIALS AND METHODS

In general, reservoir engineering uses an equation that relates surface velocity to potential gradient (Rosa et al., 2006). Such an equation is Darcy's law, which expresses the balance of momentum in a porous medium. Darcy's law is valid under a narrow set of assumptions, such as low-velocity flow (Aziz & Settari, 1990). For flow in porous media in which deviations from the classical law occur, an alternative is the proposition of a modified law, so we usually use corrections for calculating a new permeability (Li et al., 2014),

$$\mathbf{v} = -\frac{\mathbf{k}_a}{\mu}(\nabla p - \rho g \nabla D) \quad (1)$$

where $\mathbf{v}$ is the surface velocity of the fluid, $\mathbf{k}_a$ is the apparent permeability tensor (supposed to be diagonal), p is the pressure, μ is the viscosity, ρ the density, g is the magnitude of the acceleration of gravity, and D is the depth. This alternative can also be used to incorporate the slip and adsorption effects.

According to Florence et al. (2007), in certain types of flow, we must consider the occurrence of gas slippage, which is directly related to the free path of fluid molecules. The determining factor is the ratio between the mean free path of gas molecules and the characteristic length of reservoir pores. Because it has high compressibility and is subject to slippage on the solid porous medium surface, the gas flows differently from a fluid within a porous medium (Aziz & Settari, 1990). Through a correction, known as Klinkenberg correction, it is possible to incorporate the slip effect on apparent permeability, since pressure affects apparent permeability when permeability values are low (Klinkenberg, 1941). The apparent permeability, in a more general case, can also be calculated as a function of the Knudsen number (Kn) (Li et al., 2016),

$$\mathbf{k}_a = (1 + \alpha Kn) \left(\mathbf{I} + \frac{4Kn}{1 - bKn} \mathbf{I} \right) \mathbf{k} \quad (2)$$

where $\mathbf{I}$ is the identity tensor, $\mathbf{k}$ is the absolute permeability tensor, and α the rarefaction coefficient (Li et al., 2016). In the case of the slip regime, we take $\alpha=0$ and, usually, the coefficient b is equal to -1 (Li et al., 2014).

The Knudsen number measures the ratio between the mean free path of molecules, λ_n , and the characteristic pore length, R_h , so that

$$Kn = \frac{\lambda_n}{R_h} = \frac{\frac{\mu}{p} \sqrt{\frac{\pi ZRT}{2M}}}{2\sqrt{2}\tau \sqrt{\frac{k}{\phi}}} \quad (3)$$

where Z is the gas compressibility factor, R the universal gas constant, T the gas temperature, M the molecular weight of the gas, τ the tortuosity of the porous medium, ϕ the porosity of the porous medium, and k the average permeability (the geometric mean of the permeability values in the principal directions).

Depending on the Knudsen number, different flow regimes may occur:

1. Free molecular flow for $Kn \geq 10$;
2. Transition flow for $0.1 \leq Kn \leq 10$;
3. Slip regime for $10^{-3} < Kn < 0.1$;
4. Continuous for $Kn < 10^{-3}$.

Studies show that gas adsorption may be of great importance in shale gas production (Li et al., 2016, 2017). The phenomenon occurs when gas adheres to the rock surface, unlike the free gas in the pore or gas that appears dissolved in oil or water. Under high pressures, there is a significant amount of gas in the interstitial vacancies. Then, gas is released from the solid as pressure decreases (Li et al., 2016). There are two types of adsorption: physical and chemical. This work considers only the first one. In shale gas reservoirs, a significant amount of gas may be in the form of adsorbed gas (up to 80%) (Berawala, 2015).

Pressure and temperature influence the adsorption process directly. Here, we consider only the case of isothermal flow, so a Langmuir isotherm is used to model the gas adsorption (Li et al., 2016):

$$V_{ads} = \frac{pV_L}{p_L + p} \quad (4)$$

where V_{ads} is the adsorbed gas volume and V_L is the maximum adsorbed volume (Langmuir volume). Langmuir pressure, p_L , is that in which the medium adsorbs a gas volume equal to half its capacity ($0.5 V_L$).

The model presented in Jiang and Younis (2015), which incorporates adsorption in the correction of apparent permeability, is adopted here:

$$k'_a = \left[1 - \left(\frac{\frac{p}{p_L}}{1 + \frac{p}{p_L}} \right) \frac{d_m}{R_h} \right]^4 k \quad (5)$$

where d_m is the diameter of the gas molecules on the surface. Due to the effects of adsorption, we apply a new hydraulic radius (effective hydraulic radius), R_{eff} , as:

$$R_{eff} = R_h - \left(\frac{\frac{p}{p_L}}{1 + \frac{p}{p_L}} \right) d_m = R_h - \left(\frac{V_{ads}}{V_L} \right) d_m \quad (6)$$

such as $Kn' = \lambda_h / R_{eff}$. Therefore, by analogy with the previous definition of the apparent permeability, it can be extended so that it contemplates both slip and adsorption effects:

$$k_a = f(Kn') k'_a \quad (7)$$

where:

$$f(Kn') = (1 + \alpha Kn') \left(1 + \frac{4Kn'}{1 + Kn'} \right) \quad (8)$$

2.1 Governing Equations

In obtaining the governing flow equation, for unknown pressure, the following assumptions are considered:

1. The reservoir is homogeneous and anisotropic concerning intrinsic permeability;
2. Rock compressibility is small and constant;
3. No chemical reactions occur;
4. The flow is two-dimensional and occurs at low Reynolds number;
5. The flow is single-phased and isothermal;
6. The gas in the reservoir is considered real;
7. Slip and adsorption effects occur;
8. The fluid has constant chemical composition;
9. The production well is vertical and penetrates all the formation;
10. No electrokinetic effects occur;
11. There are natural fractures in the reservoir.

According to Li et al. (2016), the mass conservation for a single-phase flow in a porous medium, considering the adsorption, can be described by:

$$\frac{\partial}{\partial t} \left(\frac{\rho_{sc} \phi}{B} \right) + \frac{\partial}{\partial t} \left(\frac{\rho_s \rho_{sc} V_{ads}}{B} \right) + \nabla \cdot \left(\frac{\rho_{sc} v}{B} \right) - \frac{q_{sc} \rho_{sc}}{V_b} = 0 \quad (9)$$

where the density of the rock is ρ_s , V_b is the total volume of the rock added to the pore volume, ρ_{sc}

and q_{sc} are density and source term under standard conditions, and $B = p_{sc} Z T / T_{sc} p$ is the volume-factor-formation ($Z_{sc} \approx 1$).

Then, we introduce the modified Darcy Law (1) into the mass conservation equation (9), and we assume that the density of the rock is constant and that we can neglect the gravitational effects,

$$\nabla \cdot \left(\frac{\mathbf{k}_a}{\mu B} \nabla p \right) + \frac{q_{sc}}{V_b} = \rho_s \frac{\partial}{\partial t} \left(\frac{V_{ads}}{B} \right) + \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right) \quad (10)$$

Now, using the chain rule (Ertekin et al., 2001) and assuming small and constant compressibility,

$$\nabla \cdot \left(\frac{\mathbf{k}_a}{\mu B} \nabla p \right) + \frac{q_{sc}}{V_b} = (\Gamma_p' + \Gamma_s') \frac{\partial p}{\partial t} \quad (11)$$

where:

$$\Gamma_p' = \frac{c_\phi \phi^0}{B} + \phi \frac{d}{dp} \left(\frac{1}{B} \right) \quad (12)$$

and

$$\Gamma_s' = \rho_s \left[\frac{1}{B} \frac{dV_{ads}}{dp} + V_{ads} \frac{d}{dp} \left(\frac{1}{B} \right) \right] \quad (13)$$

where c_ϕ is the compressibility of the rock and ϕ^0 is the porosity in the pressure reference conditions (p^0).

In this work, we simulate a two-dimensional flow in cylindrical geometry. Then, from Eq. (11), it is possible to write the equation that is solved numerically, knowing that the apparent permeability tensor is diagonal and that we impose the flow rate in the well via an internal boundary condition:

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{k_{ar}}{\mu B} \frac{\partial p}{\partial r} \right) V_b + \frac{\partial}{\partial z} \left(\frac{k_{az}}{\mu B} \frac{\partial p}{\partial z} \right) V_b = \\ = (\Gamma_p' + \Gamma_s') V_b \frac{\partial p}{\partial t} \end{aligned} \quad (14)$$

Equation (14) is a nonlinear partial differential equation that we use to determine reservoir pressure. Its resolution requires the proposition of the appropriate initial and boundary conditions. As an initial condition, we consider for $t=0$,

$$p(r, z, t = 0) = p_{ini}(r, z) = p_{ini} \quad (15)$$

It represents the initial pressure before the reservoir is disturbed by production/injection.

We impose null flow conditions at the outer boundaries,

$$\left(\frac{\partial p}{\partial r} \right)_{r=L_r} = \left(\frac{\partial p}{\partial z} \right)_{z=0, L_z} = 0 \quad (16)$$

where L_r and L_z are the reservoir lengths in the r - and z - directions, respectively, and for the inner boundary,

$$\left(\frac{\partial p}{\partial r} \right)_{r=r_w} = - \frac{q_{sc} \mu B}{2\pi h_{layer} k_{ar} r_w} \quad (17)$$

which represents the flow entering the producing well from the layer of thickness h_{layer} , and r_w is the well radius. This term accounts for gas production and we introduce (Lee & Wattenbarger, 1996)

$$q_{sc} = -J_w(p - p_{wf}) \quad (18)$$

where J_w is the productivity index and p_{wf} is the pressure in the producing well. As the well traverses all the extension of the reservoir, the total production flow is given by:

$$Q_{sc} = \Sigma q_{sc} \quad (19)$$

depending on the different layers of the reservoir in contact with the well.

2.2 Gas properties

To use the model presented, we introduce now the fluid properties. We start with γ , which is the ratio of gas molecular weight, M , to air molecular weight, M_{air} ,

$$\gamma = \frac{M}{M_{air}} = \frac{M}{28.96} \quad (20)$$

For gas mixtures of hydrocarbons, the pseudo-critical pressure and temperature coordinates, p_{pc} and T_{pc} , are used to determine the so-called pseudo-reduced coordinates, which are utilized to calculate the physical properties of natural gas (Rosa et al., 2006). Taking the study of Sutton (1985) as a reference, we employ the following correlations in obtaining pseudo-critical pressure

$$p_{pc} = 756.8 - 131\gamma - 3.6\gamma^2 \quad (21)$$

and pseudo-critical temperature:

$$T_{pc} = 169.2 - 349.5\gamma - 74\gamma^2 \quad (22)$$

The compressibility factor Z acts as a correction for the deviation of behavior between an ideal gas and a real gas. Therefore, Z is the ratio between the volume that a given gas mass occupies in reservoir conditions and the volume that the same mass would occupy if it were an ideal gas (Rosa et al., 2006). According to Sanjari and Lay (2011), the gas compressibility factor can be calculated using the correlation:

$$Z = 1 + A_1 p_{pr} + A_2 (p_{pr})^2 + A_3 \frac{(p_{pr})^{A_4}}{(T_{pr})^{A_5}} + A_6 \frac{(p_{pr})^{A_4+1}}{(T_{pr})^{A_7}} + A_8 \frac{(p_{pr})^{A_4+2}}{(T_{pr})^{A_7+1}} \quad (23)$$

where p_{pr} and T_{pr} are pseudo-reduced pressure and temperature, respectively (Lee et al., 1966); $A_1 = 0.007698$; $A_2 = 0.003839$; $A_3 = -0.467212$; $A_4 = 1.018801$; $A_5 = 3.805723$; $A_6 = -0.087361$; $A_7 = 7.138305$; and $A_8 = 0.083440$ for $0.01 \leq p_{pr} \leq 3.0$. Now, for $3.0 \leq p_{pr} \leq 15.0$, $A_1 = 0.015642$; $A_2 = 0.000701$; $A_3 = 2.341511$; $A_4 = -0.657903$; $A_5 = 8.902112$; $A_6 = -1.136000$; $A_7 = 3.543614$; and $A_8 = 0.134041$.

Using the values obtained from Z and the universal gas constant, R , it is possible to determine the density from the state equation for a real gas, $\rho = pM/ZRT$.

Natural gas viscosity can be calculated using the correlation suggested by Lee et al. (1966), widely used in reservoir simulation (Ertekin et al., 2001),

$$\mu = K \times 10^{-4} \exp(X\rho_g^Y) \quad (24)$$

where K , X , Y , and ρ_g depend on T and M , and we can find its expressions in Lee et al. (1966).

3. THE LAYER MODEL

Over the years, several models have been proposed to represent the flow in naturally fractured media. In this work, we adopt the layer model (Warren & Root, 1963; Kazemi, 1969), in which we alternate in layers, in the solution domain, the regions of porous matrix and fracture. Each fracture layer has higher permeability and porosity than matrix layers, and the total volume occupied by fractures is smaller than that occupied by the matrix. In this model, there are different

configuration possibilities. These possibilities depend on the coordinate system and the distribution of fractures in the matrix (Bourdet, 2002).

In the naturally fractured system, a dual-porosity model assumes that the matrix does not produce directly into the well and that the flow comes from the fracture to the well. If we accept that the flow comes from the matrix and fracture to the well, we have a dual-permeability model. Flow is considered restricted if there is a block between matrix and fracture (Bourdet, 2002).

We should consider two dimensionless parameters when it comes to using the layer model: the storage ratio coefficient, represented by ω , and the pore flow coefficient, represented by λ . The storage coefficient defines the contribution of the fracture system to the total storage, generally ranging from 10^{-3} to 10^1 . The flow coefficient defines the production capacity of the matrix blocks for the fracture, denoting the communication between the fracture-matrix system.

When λ is low, it is difficult to produce from the matrix to wellbore, causing the system to take longer to behave like the equivalent of a homogeneous system. The coefficient λ usually ranges from 10^{-10} to 10^{-4} . We can find formulas for λ and ω for different configurations in Bourdet (2002).

Finally, it is worth mentioning that when production begins in a naturally fractured reservoir, the flow has four regimes (Bourdet, 2002):

1. At first, production mainly occurs through flow in fractures;
2. Then, after a while, a transition period occurs;
3. In the third period, fractured and non-fractured regions contribute to the production;
4. Following, the boundary effects appear.

Figure 3 illustrates the configuration adopted in this work, for rz geometry with a normal z fracture, and for a dual-permeability model and unrestricted flow.

Figure 3. Layer model: well, matrix (in gray), and fracture (in black).

Figure 4. Two-dimensional rz discretization.

3.1 Discretization

In the physical domain partitioning, we use a computational grid of centered blocks (Aziz & Settari, 1990; Ertekin et al., 2001; Abou-Kassem et al., 2006; Chen et al., 2006) in cylindrical coordinates. In Figure 4, we see the schematic representation of the discretized two-dimensional domain for the rz cylindrical coordinate system, where the fracture thickness is h_f , and the reservoir thickness is equal to the sum of the matrix

thickness (h_m) plus the fracture thickness. From the computational mesh nodes (positioned in the center of the cells), we obtain the numerical solution, and n_r and n_z represent, respectively, the number of cells in the r - and z - directions. The i index represents cell numbering in the r -direction while k symbolizes cell numbering in the z -direction. We indicate the faces of the cells (blocks) in the computational mesh by the fractional indexes $i \pm 1/2$ and $k \pm 1/2$.

The discrete form of the governing equation, in the cell (i, k) and time $n+1$, is given by:

$$\begin{aligned} \left[(\Gamma_p + \Gamma_s) \frac{\partial p}{\partial t} \right]_{i,k}^{n+1} &= \\ &= \left[\frac{1}{r} \frac{\partial}{\partial r} \left(T_r \frac{\partial p}{\partial r} \right) r dr \right]_{i,k}^{n+1} + \left[\frac{\partial}{\partial z} \left(T_z \frac{\partial p}{\partial z} \right) dz \right]_{i,k}^{n+1} \end{aligned} \quad (25)$$

where the original equation was multiplied by the cell volume (i, k) so that the coefficients can already incorporate the geometric properties of the mesh: $\Gamma_p = \Gamma'_p V_b$ and $\Gamma_s = \Gamma'_s V_b$ (Ertekin et al., 2001).

Employing space centered difference approximations commonly used in reservoir simulation (Ertekin et al., 2001), we have:

$$\begin{aligned} \frac{\partial}{\partial r} \left(T_r \frac{\partial p}{\partial r} \right)_{i,k}^{n+1} &\approx \\ &\approx \frac{1}{\Delta r_{i,k}} \left[\left(T_r \frac{\partial p}{\partial r} \right)_{i+1/2,k} - \left(T_r \frac{\partial p}{\partial r} \right)_{i-1/2,k} \right]^{n+1} \end{aligned} \quad (26)$$

and

$$\begin{aligned} \frac{\partial}{\partial z} \left(T_z \frac{\partial p}{\partial z} \right)_{i,k}^{n+1} &\approx \\ &\approx \frac{1}{\Delta z_{i,k}} \left[\left(T_z \frac{\partial p}{\partial z} \right)_{i,k+1/2} - \left(T_z \frac{\partial p}{\partial z} \right)_{i,k-1/2} \right]^{n+1} \end{aligned} \quad (27)$$

Using, again, centered difference approximations and considering only the r -direction,

$$\left(\frac{\partial p}{\partial r} \right)_{i+1/2,k}^{n+1} \approx \frac{p_{i+1,k}^{n+1} - p_{i,k}^{n+1}}{\Delta r_{i+1/2,k}} \quad (28)$$

and

$$\left(\frac{\partial p}{\partial r} \right)_{i-1/2,k}^{n+1} \approx \frac{p_{i,k}^{n+1} - p_{i-1,k}^{n+1}}{\Delta r_{i-1/2,k}} \quad (29)$$

where $\Delta r_{i\pm 1/2,k}$ is the distance between cell nodes (i, k) and $(i\pm 1, k)$. Similarly, we obtain approximations for the derivatives in the z -direction.

Due to the value of the pressure gradient near the well, we use a non-uniform mesh in the r -direction (Ertekin et al., 2001). We have omitted the k index for simplicity of notation.

$$\delta_r = \left(\frac{r_e}{r_w} \right)^{1/n_r} \quad (30)$$

where r_e is the outer radius of the reservoir, n_r is the number of cells in the r -direction, and δ_r is a coefficient such that:

1. From the well radius, we space the nodes according to the ratio

$$r_{i+1} = \delta_r r_i = \left(\frac{r_e}{r_w} \right)^{1/n_r} r_i \quad (31)$$

where $i=1,2,\dots,n_r-1$;

2. Cell boundaries are defined by

$$r_{i+1/2} = \frac{r_{i+1} - r_i}{\ln \left(\frac{r_{i+1}}{r_i} \right)} \quad (32)$$

where $i=1,2,\dots,n_r-1$; and

3. In determining cell volumes,

$$r_{i+1/2}^2 = \frac{r_{i+1}^2 - r_i^2}{\ln \left(\frac{r_{i+1}}{r_i} \right)} \quad (33)$$

In the case of the z -direction, we also make progressive mesh refinement from the fracture location (de Souza et al., 2010; de Souza, 2013),

$$\delta_z = \frac{1}{n_{CZ}} \ln \left(\frac{z_{max}}{z_{min}} \right) \quad (34)$$

where for n_z always odd,

$$z_{min} = \frac{h_f}{2} \quad (35)$$

$$z_{max} = \frac{h_m + h_f}{2} \quad (36)$$

$$n_{CZ} = \frac{n_z - 1}{2} \quad (37)$$

Such as, for $k < n_{CZ}$,

$$\Delta z_{i,k} = z \{ \exp[(n_{CZ} - k + 1)\delta_z] - \exp[(n_{CZ} - k)\delta_z] \}_{min} \quad (38)$$

for $k = n_{CZ}$,

$$\Delta z_{i,k} = h_f \quad (39)$$

and for $k > n_{CZ}$

$$\Delta z_{i,k} = z\{exp(k\delta_z) - exp[(k-1)\delta_z]\}_{min} \quad (40)$$

3.2 Numerical formulation

Continuing with our development, we introduce the transmissibilities:

$$T_{r,i\pm 1/2,k}^{n+1} = \left(\frac{G_r}{\mu B}\right)_{i\pm 1/2,k}^{n+1} \quad (41)$$

and

$$T_{z,i,k\pm 1/2}^{n+1} = \left(\frac{G_z}{\mu B}\right)_{i,k\pm 1/2}^{n+1} \quad (42)$$

Considering that the total angle is 2π , and using the apparent permeability (Ertekin et al., 2001),

$$G_{r,i+\frac{1}{2},k}^{n+1} = \frac{\pi \Delta z_k}{\left(\frac{1}{k_{ar,i,k}}\right)^{n+1} \ln\left(\frac{r_{i+1/2}}{r_i}\right) + \left(\frac{1}{k_{ar,i+1,k}}\right)^{n+1} \ln\left(\frac{r_{i+1}}{r_{i+1/2}}\right)} \quad (43)$$

and

$$G_{z,i,k+1/2}^{n+1} = \frac{\pi(r_{i+1/2}^2 - r_{i-1/2}^2)}{\frac{z_{k+1/2} - z_k}{k_{az,i,k}^{n+1}} + \frac{z_{k+1} - z_{k+1/2}}{k_{az,i,k+1}^{n+1}}} \quad (44)$$

using a weighted average to determine the fluid properties at the interfaces (Ertekin et al., 2001).

When it comes to the term of accumulation, $(\Gamma_p + \Gamma_s) \partial p / \partial t$, conservative expansion is used to determine the multiplicative coefficients (Ertekin et al., 2001),

$$\Gamma_{p,i,k}^{n+1} = V_{i,k} \left[\frac{1}{B^n} \frac{d\phi}{dp} + \phi^{n+1} \frac{d}{dp} \left(\frac{1}{B} \right) \right]_{i,k} \quad (45)$$

and

$$\Gamma_{s,i,k}^{n+1} = V_{i,k} \left[\rho_s V_{ads}^n \frac{d}{dp} \left(\frac{1}{B} \right) + \frac{\rho_s}{B^{n+1}} \frac{dV_{ads}}{dp} \right]_{i,k} \quad (46)$$

where $V_{i,k}$ is the volume of the cell (i, k) , Δt is the time increment. We employ a backward difference approximation for the derivative concerning the time

$$\left(\frac{\partial p}{\partial t} \right)_{i,k}^{n+1} \approx \frac{p_{i,k}^{n+1} - p_{i,k}^n}{\Delta t} \quad (47)$$

resulting in a fully implicit formulation.

Following the strategy used by Tavares (2003) and Li et al. (2016) for the case without storage, we have

$$Q_{sc} = \sum_{k=1}^{k=n_z} q_{sc1,k}^{n+1} \quad (48)$$

where the term $q_{sc1,k}^{n+1}$ is the contribution of the cell $(1, k)$ in contact with the well to the production flow rate. According to Tavares (2003),

$$q_{sc1,k}^{n+1} = -J_{w1,k}^{n+1} (p_{1,k}^{n+1} - p_{wf,k}^{n+1}) \quad (49)$$

where the productivity index J_w (Lee & Wattenbarger, 1996), incorporating apparent permeability, is given by:

$$J_{w1,k}^{n+1} = \left(\frac{k_{ar}}{\mu} \right)_{1,k}^{n+1} \frac{\Delta z_{1,k} T_{sc}}{p_{sc} \ln \left(\frac{r_o}{r_w} \right) T} \quad (50)$$

where:

$$r_o = \sqrt{r_{i+1/2} r_w} \quad (51)$$

3.3 System of equation solution

Aiming to obtain a system of linearized equations, we use the Picard method (Nick et al., 2013),

$$\begin{aligned} \frac{(\Gamma_p + \Gamma_s)_{i,k}^{n+1,v}}{\Delta t} p_{i,k}^n = & -T_{z|i,k-1/2}^{n+1,v} p_{i,k-1}^{n+1,v+1} - T_{r|i-1/2,k}^{n+1,v} p_{i-1,k}^{n+1,v+1} \\ & + \left[T_{z|i,k-1/2}^{n+1,v} + T_{r|i-1/2,k}^{n+1,v} + T_{z|i,k+1/2}^{n+1,v} + T_{r|i+1/2,k}^{n+1,v} + \frac{(\Gamma_p + \Gamma_s)_{i,k}^{n+1,v}}{\Delta t} \right] p_{i,k}^{n+1,v+1} \\ & - T_{z|i,k+1/2}^{n+1,v} p_{i,k+1}^{n+1,v+1} - T_{r|i+1/2,k}^{n+1,v} p_{i+1,k}^{n+1,v+1} \end{aligned} \quad (52)$$

where v refers to the current iterative level, and the properties calculated at that level are used to determine the pressures at $v+1$, the next iterative level.

Figure 5 illustrates the nonzero elements of the coefficient matrix, which corresponds to a pentadiagonal pattern plus an extra column and row due to well-reservoir coupling (well pressure is also unknown to the problem).

Figure 5. Coefficient matrix.

After the linearization process of the nonlinear algebraic equations, we choose the Preconditioned Conjugate Gradient (PCG) (Saad, 2003) iterative method to obtain the linear system solution. According to Ertekin et al. (2001), when dealing with strong heterogeneities, approximating factorization methods are recommended rather than using stationary iterative methods (such as Gauss-Seidel and SOR). Here we adopt the preconditioner of Jacobi (Ertekin et al., 2001; Saad, 2003).

In Figure 6, we can see the flowchart for the numerical solution algorithm for a time step for the pressure calculation. The numeric code was developed in the C programming language, using GIT for version control and Eclipse IDE.

Figure 6. Solution for a single time step.

4. NUMERICAL RESULTS

We present the results in two types of graphs. In specialized, we show the wellbore pressure as a function of time. In turn, the diagnostic plot shows well pressure variation (Δp_{wf}) in continuous line and its Bourdet derivative ($\Delta p'_{wf}$) in a dashed line (Bourdet et al., 1989) as a function of time. Through the analysis of these graphs, it is possible to identify flow regimes for the well-reservoir coupling. There are different formulas for implementing the Bourdet derivative. Here, we adopt the three-point scheme (New Mexico Tech, 2019).

Table 1. Parameters for the default case.

Parameter	Value	Unit
C_ϕ	1.0×10^{-6}	psi
d_m	2.3×10^{-10}	ft
h_f	0.05	ft
k_f	0.05	Darcy
k_r and k_z	8.0×10^{-6}	Darcy
L_r	1,500	ft
L_z	40.0	ft
L_{wf}	40.0	ft
n_r	32	–
n_z	15	–
p_{sc}	14.65	psi
p_L	1,100	psi
p_{ini} and p^0	4,500	psi
Q_{sc}	-1.0×10^4	scf/day
r_w	0.25	ft
R	10.73	ft psi/R lbm-mol
t_{max}	200	day
tol	1.0×10^{-5}	psi
T	609.67	R
T_{sc}	519.67	R
V_L	0.0005	ft/lbm
$\delta_{\Delta t}$	1.2	–
Δt_{ini}	0.0001	day
Δt_{max}	10.0	day
ρ_s	200.0	lbm/ft
τ	1.41	–
ϕ_f	0.60	–
ϕ_{ini} and ϕ^0	0.12	–

$$\left(\frac{d\Delta p}{d \ln \Delta t}\right)_j = \frac{\Delta p_j - \Delta p_{j-1}}{\ln\left(\frac{\Delta t_j}{\Delta t_{j-1}}\right)} \frac{\ln\left(\frac{\Delta t_{j+1}}{\Delta t_j}\right)}{\ln\left(\frac{\Delta t_{j+1}}{\Delta t_{j-1}}\right)} + \frac{\Delta p_{j+1} - \Delta p_j}{\ln(\Delta t_{j+1}/\Delta t_j)} \frac{\ln(\Delta t_j/\Delta t_{j-1})}{\ln(\Delta t_{j+1}/\Delta t_{j-1})}, j \geq 2 \quad (53)$$

However, it is noteworthy that there are two-point versions with data filtering, so that not all data are considered, to eliminate noise when calculating the derivative from field data (Bourdet et al., 1989).

Initially, we performed simulations using a standard data group, Table 1, based on the use of the non-Darcy model of Jiang and Younis (2015) in a naturally fractured reservoir, where we incorporated the slip and adsorption effects in apparent permeability.

Through an arrangement consisting of a vertical producer well (of length L_{wf} and centered on the $r\theta$ -plan), the reservoir produces gas until we reach the maximum production time (t_{max}).

The time increment is variable and starts with a stipulated initial value (Δt_{ini}) that is incremented using the growth ratio ($\delta_{\Delta t}$) until it reaches the maximum prescribed value (Δt_{max}). This procedure is commonly used in reservoir simulation when it is necessary to evaluate the wellbore pressure in the initial moments (order of seconds) when there are more pronounced pressure gradients (Ertekin et al., 2001).

In this work, we do not consider gravitational effects, rarefaction, storage, and damage to the formation. The values of the physical parameters were chosen from the Li et al.'s (Li et al., 2016) article for shale gas and from the work of de Souza (2013), which also simulated gas flow in cylindrical coordinates using a well-reservoir coupling.

4.1 Numerical verification

We performed a mesh refinement study, and the number of cells used in the generation of the different computational meshes is available in Table 2. Figures 7 and 8 show the results in the specialist and diagnostic plots, respectively, for the four different meshes with a maximum production time of 200 days. Unless otherwise noted, we employ non-uniform meshes in r - and z - directions, as explained in Section 3.1.

Table 2. Mesh refinement.

Mesh	n_r	n_z
1	8	7
2	16	11
3	32	15
4	64	19

From the figures, we observe numerical convergence as meshes are refined by increasing the number of cells in the r - and z - directions. The curves are practically overlapping. Therefore, considering that numerical convergence was verified, we decided to use Mesh 3 ($n_r = 32$ and $n_z = 15$) in all subsequent simulations, due to the lower computational effort when compared to Mesh 4.

In Figure 7, we can observe the existence of two typical regimes for this type of flow, disregarding the storage effect. First, a part directly related to the transient flow regime, represented by a straight line in the specialized graph, and, then, a second part, shortly after the transition regime, which denotes the pseudo-radial flow, typical of naturally fractured reservoirs. The different regimes can be distinguished more easily by looking at the diagnostic plot in Figure 8, where we see, in the middle, the transition regime linking the two others. This regime represents the effect of the transition of flow coming from the fracture and the matrix-fracture.

We also studied the influence of tolerance criteria to stipulate the convergence of internal and external iterative processes (Figure 6), in the variation of the pressure field. Three tolerance

Figure 7. Specialized plot for mesh refinement. **Figure 8.** Diagnostic plot for mesh refinement.

Figure 9. Specialized plot for tolerance variation.

Figure 10. Diagnostic plot for tolerance variation.

values $tol=10^{-4}$, 10^{-5} , and 10^{-6} psi were employed, and the results found are shown in Figures 9 and 10 for the specialized and diagnostic plots, respectively. There were no significant differences between the values determined with the different stopping criteria. Therefore, we kept the default value provided in Table 1 for all other simulations.

4.2 Sensitivity analysis for fracture properties

After the first tests, we performed a sensitivity study based on the variation of some of the main physical parameters that are relevant to determine the effects of the presence of fracture in the well-reservoir and matrix-fracture system. First, we evaluate the effect of changing the value of the permeability of the fracture on the flow, Figures 11 and 12.

The results presented include the slip and adsorption effects via the variation of apparent permeability. Since the Knudsen number is inversely proportional to the characteristic pore length (which in turn is directly proportional to permeability), a decrease in permeability will increase the Knudsen number and apparent permeability. From the analysis of the results, we see the evident change in the behavior of the matrix-fracture system as a consequence of changing k_f .

As expected, the transition regime depends straightly on the fracture permeability value. Thus, as the fracture permeability value decreases, the system becomes less heterogeneous, and the transition regime becomes shorter. Such behavior is evidenced by the Bourdet derivative so that for higher permeability values, the boundary effects

 Figure 11. Specialized plot for k_f variation.

 Figure 12. Diagnostic plot for k_f variation.

 Figure 13. Specialized plot for ϕ_f variation.

are felt earlier with the consequent change in the slope of the Bourdet derivative curve for the final moments of production.

As we did with permeability, we changed

fracture porosity values (ϕ_f) and performed simulations using values 0.4, 0.6 (default), and 0.8. Specialized graph of Figure 13 and the diagnostic plot of Figure 14 show the results. The fracture

Figure 14. Diagnostic plot for ϕ_f variation.

porosity value also modifies the Knudsen number, but its effect is the opposite of that of permeability. From the analysis of the pressure curves, we observe that the higher the porosity of the fracture is, the lower the pressure drop will be. For production with a constant-rate flow (our case), this is due to the higher amount of fluid available. Unlike in the case of permeability variation, we observe no significant change concerning flow regimes.

While making comparisons for different values of the fracture thickness for the tested values 0.025 ft, 0.05 ft (default), and 0.1 ft, we find that the fracture thickness has a significant influence on the flow regimes and pressure drop in the well. For the highest thickness values, because the fracture has higher permeability and porosity than the

matrix, a preferential flow region appears. This led to higher fluid production and lower pressure drop. We can confirm these findings from the visualization of Figures 15 and 16. We can also observe the effect that the thickness h_f has on the flow regimes, before and after the transition regime. These regimes characterize the production coming from the fracture and matrix-fracture, respectively.

4.3 Sensitivity analysis for general properties

We begin with the study of the implications of reservoir temperature modification on pressure variation. We take temperatures equal to 559.67 R, 609.67 R (default), and 659.67 R. Temperature is known to modify parameters affecting the flow,

Figure 15. Specialized plot for h_f variation.

 Figure 16. Diagnostic plot for h_f variation.

 Figure 17. Specialized plot for T variation.

 Figure 18. Diagnostic plot for T variation.

such as gas viscosity and Knudsen number. In Figures 17 and 18, for the specialized and diagnostics plots, we can see that pressure varies faster for the higher temperature values, which, in

the case of gas, is a consequence of the fact that viscosity increases with temperature. The higher the viscosity is, the greater the pressure gradient must be for a constant-rate flow (Darcy's law).

Figure 19. Specialized plot for V_L variation.

Figure 20. Diagnostic plot for V_L variation.

Nevertheless, the higher the temperature is, the higher the number of Knudsen will be. Thus, there is an overlap of effects, with the viscosity variation becoming more prevalent as temperature increases.

When performing the tests varying the volume of Langmuir ($V_L = 2.5 \cdot 10$, $5.0 \cdot 10$ and $7.5 \cdot 10$ ft³/lbm), Figures 19 (specialized) and 20 (diagnostic), we note that higher values of V_L lead to a lower drop for the tested range. We assume that this behavior is due to the adsorption effect, which contributes to a higher amount of fluid available for production. The reservoir surface releases the gas as the pressure in the reservoir decreases and, consequently, the apparent permeability also changes as a function of adsorption effects. Moreover, when analyzing the diagnostic plot, we note that the flow transition regime takes longer to

occur. The late appearance of the pseudo-radial flow regime is a consequence of this behavior when compared to the results for the lowest values of V_L . Besides, the characteristic bilinear regime lasts less time because the adsorbed gas is being released, contributing to the maintenance of pressure.

Regarding Langmuir pressure (pressure at which adsorption reaches half of its maximum capacity), we do not obtain noticeable variations for the values 990 psi, 1,100 psi, and 1,210 psi. These results contrast with the previous ones, when we vary the volume of Langmuir (although the percentage of variation is quite different). By analyzing the graphs of Figures 21 and 22, it is possible to ponder that the curves are practically overlapping. This may happen due to the combined effects of the bilinear flow, slip, and adsorption

Figure 21. Specialized plot for p_L variation.

Figure 22. Diagnostic plot for p_L variation.

effects that balance the pressure drop. In this case, the solid surface of the porous medium does not release the trapped gas.

Finally, aiming to highlight the effects of slip and adsorption on the single-phase gas flow in a fractured reservoir, four cases were considered:

1. Case 1: Classic Darcy Law;
2. Case 2: Modified Darcy's law considering only the slip effects on apparent permeability;
3. Case 3: Modified Darcy's law considering only the slip effects on apparent permeability and a source term for adsorption effects;
4. Case 4: Modified Darcy's law considering slip and adsorption effects on apparent permeability and a source term for adsorption effects.

Initially, we utilize the specialized graph, Figure 23, to analyze the results. Over time, Case 1 and 2 have the pressure drop more pronounced. However, Case 2 shows a lower pressure drop for times longer than ten days. In the first case, there are no effects that can reduce the pressure drop over time. In Case 2, the slip effect implies a slight reduction in pressure drop compared to Case 1 as time progresses. When the effects of slip and adsorption (not affecting apparent permeability) are combined, Case 3, they promote a noticeable reduction in pressure drop. However, we see the most noticeable change in Case 4, where slip and adsorption effects (now correcting apparent permeability) act together to retard pressure drop more significantly.

Then, when analyzing the diagnostic plot of Figure 24, the differences between the flow

Figure 23. Specialized plot for different models.

Figure 24. Diagnostic plot for different models.

regimes for the four cases can be seen in more detail. We can observe how each effect separately, or together, modifies the transition regime and the time when boundary effects appear.

In all tests performed, the Knudsen number kept its value below 10, ensuring that it was outside the molecular flow range.

To the knowledge of the authors, there is no work applying the layer model incorporating the effects of slipping and adsorption for flow in shale gas reservoirs. In some studies, such as [Guo et al. \(2015\)](#) and [Tao et al. \(2018\)](#), the authors applied other techniques and geometries, and their results were obtained for fractured reservoirs and also included the effects of slipping and adsorption. From their results, it is possible to detect qualitative similarities with our results concerning the effects of the presence of fractures, pressure

variation in the well, and the Bourdet derivative, after wellbore storage effects, which we did not consider in this work.

5. CONCLUSIONS

The model employed was able to capture successfully the different regimes associated with single-phase flow in fractured shale gas reservoirs. Therefore, in future applications, we could use the simulator as a useful tool to determine ideal conditions aiming at increasing production, in a scenario that presents the greatest financial return, if we perform simulations considering real reservoirs. The simulation of more realistic conditions would consider three-dimensional Cartesian coordinates and the use of horizontal wells (even hydraulically fractured), aiming to have

a shale gas reservoir producing in an economically viable way. However, the purpose of this article was to perform a numerical well pressure test.

The tests showed the relevance of using a complete model, considering the slip and adsorption effects, in the case of the gas flow in reservoirs with low permeability and containing fractures.

We also saw that, in the specific case investigated, the non-consideration of these phenomena, mainly the adsorption, can lead to predictions that do not match the reality. The production of shale reservoirs is made possible by taking these effects into account. This also leads to maintaining wells that have a longer producing life. It is noteworthy that the effect of the fracture overlapped the slip at the beginning of the flow, favorably contributing to a decrease in resistance to gas flow.

However, the use of horizontal wells and hydraulic fracturing are also important to increase the production in these reservoirs. Future research produced by our group should include these features.

6. REFERENCES

- Abou-Kassem, J. H.; Farouq Ali, S. M.; Islam, M. R. **Petroleum Reservoir Simulation, A Basic Approach**. Houston, USA: Gulf Publishing Company, 2006.
- Ali, W. **Modeling gas production from shales and coal-beds**. Master's thesis, Stanford University, 2012.
- Aziz, M.; Settari, A. **Petroleum Reservoir Simulation**. New York: Elsevier Applied Science, 1990.
- Berawala, D. S. **Modeling of gas production from tight shale formations: an innovative approach**. Master's thesis, University of Stavanger, Stavanger, Norway, 2015.
- Bourdet, D. **Well Test Analysis: The Use of Advanced Interpretation Models**. Handbook of Petroleum Exploration and Production 3. Amsterdam: Elsevier, 2002.
- Bourdet, D.; Ayoub, J. A.; Pirard, Y. M. Use of pressure derivative in well-test interpretation. **SPE Formation Evaluation**, v. 4, n. 2, p. 293-302, 1989. <https://doi.org/10.2118/12777-PA>
- Chen, Z.; Huan, G.; Ma, Y. **Computational Methods for Multiphase Flows in Porous Media**. Philadelphia, USA: Society of Industrial and Applied Mathematics, 2006. <https://doi.org/10.1137/1.9780898718942>
- Chung, C.; de Souza, G.; Amaral Souto, H. Simulação de reservatórios de óleo leve com poço horizontal hidraulicamente fraturado. **Revista Brasileira de Engenharia e Física Aplicada**, v. 2, n. 2, p. 1-16, 2017. <https://doi.org/10.17648/calibre.v2i2.196>
- de Souza, G. **Acoplamento Poço-reservatório na Simulação Numérica de Reservatórios de Gás**. PhD Thesis, Universidade Estadual do Norte Fluminense, Macaé, Brasil, 2013. (in Portuguese)
- de Souza, G.; Amaral Souto, H. Numerical reservoir simulation of naturally fractured reservoirs. **Proceedings of the XXXV Iberian Latin-American Congress on Computational Methods in Engineering**, Fortaleza, Brazil, 23-26 November 2014.
- de Souza, G.; Zacchi, A.; Barreto, A. Pires, A. Analysis of the effect of different Forchheimer's beta correlations in the results of numerical simulation of gas flow in naturally fractured reservoir. **Proceedings of the Latin America and Caribbean Exhibition and Conference**, Lima, Peru, 1-3 December, 2010. <https://doi.org/10.2118/139175-MS>
- Debossam, J. G. S.; Heringer, J. D. S.; de Souza, G.; Souto, H. P. A. Numerical simulation of single-phase flow in naturally fractured oil reservoirs. **Coupled Systems Mechanics**, v. 8, n. 2, p. 129-146, 2019. <https://doi.org/10.12989/csm.2019.8.2.129>
- Ertekin, T.; Abou-Kassem, J. H.; King, G. R. **Basic Applied Reservoir Simulation**. SPE Textbook Series 7. Society of Petroleum Engineers, Richardson, 2001.
- Fekete Associates. **Fekete well test**. Available at <http://www.fekete.ca/software/welltest/Pages/About-FAST-WellTest.aspx>. Accessed on May, 7th, 2019.

Florence, F. A.; Rushing, J. A.; Newsham, K. E.; Blasingame, T. A. Improved permeability prediction relations for low permeability sands. **Society of Petroleum Engineers, Rock Mountain Oil & Gas Technology Symposium**, Denver, Colorado, USA, 16-18 April 2007. <https://doi.org/10.2118/107954-MS>

Gomes, M. J. **Estudo do mercado brasileiro de gás natural contextualizado ao shale gas**. Technical report, Universidade Federal do Rio Grande do Sul, 2011.

Guo, B.; Ghalambor, A. **Natural Gas Engineering Handbook**. Elsevier, 2014.

Guo, C.; Xu, J.; Wei, M.; Jiang, R. Pressure transient and rate decline analysis for hydraulic fractured vertical wells with finite conductivity in shale gas reservoirs. **Journal of Petroleum Exploration and Production Technology**, v. 5, p. 435-443, 2015. <https://doi.org/10.1007/s13202-014-0149-3>

Hasan, S.; Farooqui, M.; Raoand, P.; Ramachandran, K.; Tripathy, P.; Harinarayana, T. Petroliferous basins and shale gas – an unconventional hydrocarbon asset of India. **Geosciences**, v. 3, n. 4, p. 108-118, 2013.

Heinemann, Z. E.; Mittermeir, G. **Natural Fractured Reservoir Engineering**. PHDG Association, Tehran, 2014.

Ishimoto, K. Improved matrix, fracture fluid transfer function in dual porosity models. **International Meeting on Petroleum Engineering**, Tianjin, China, 1-4 November 1988. <https://doi.org/10.2118/17599-MS>

Jacomo, J. C. P. **Os hidrocarbonetos não convencionais: Uma análise da exploração do gás de folhelho na Argentina à luz da experiência norte-americana**. Dissertação de mestrado, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2014. (in Portuguese)

Jia, B.; Tsau, J.-S.; Barati, R. A workflow to estimate shale gas permeability variations during the production process. **Fuel**, v. 220, p. 879-889, 2018. <https://doi.org/10.1016/j.fuel.2017.11.087>

Jiang, J.; Younis, R. M. A multimechanistic multicontinuum model for simulating shale gas reservoir with complex fractured system. **Fuel**, v. 161, p. 333-344, 2015. <https://doi.org/10.1016/j.fuel.2015.08.069>

Kappa Engineering. Kappa. Available at <https://www.kappaeng.com/>. Accessed on June, 10th, 2018.

Kazemi, H. Pressure transient analysis of naturally fractured reservoirs with uniform fracture distribution. **SPE Journal**, v. 9, n. 4, p. 451-462, 1969. <https://doi.org/10.2118/2156-A>

Klinkenberg, L. J. The permeability of porous media to liquids and gases. **Drilling and Production Practice**, American Petroleum Inst., New York, NY, January 1st, 1941.

Lee, A.; Gonzalez, M.; Eakin, B. The viscosity of natural gases. **Journal of Petroleum Technology, Transactions of AIME**, v. 18, n. 8, p. 997-1000, 1966. <https://doi.org/10.2118/1340-PA>

Lee, W.; Wattenbarger, R. **Gas Reservoir Engineering**. SPE textbook series. Society of Petroleum Engineers, 1996.

Li, D.; Xu, C.; Wang, J.; Lu, D. Effect of Knudsen diffusion and Langmuir adsorption on pressure transient response in tight- and shale-gas reservoirs. **Journal of Petroleum Science and Engineering**, v. 124, p. 146-154, 2014. <https://doi.org/10.1016/j.petrol.2014.10.012>

Li, D.; Zhang, L.; Wang, J. Y.; Lu, D.; Du, J. Effect of adsorption and permeability correction on transient pressures in organic rich gas reservoirs: Vertical and hydraulically fractured horizontal wells. **Journal of Natural Gas Science and Engineering**, v. 31, p. 214-225, 2016. <https://doi.org/10.1016/j.jngse.2016.02.033>

Li, Y.; Dong, P.; Li, W.; Mi, L.; Yang, S.; Cao, N.; Yi, X. The gas flow in micro fractures of shale gas reservoirs. **SPE Middle East Oil & Gas Show and Conference**, Manama, Kingdom of Bahrain, 6-9 March 2017. <https://doi.org/10.2118/183972-MS>

Nelson, R. **Geologic Analysis of Naturally Fractured Reservoirs**. Gulf Professional Publishing, USA, Second edition, 2001. <https://doi.org/10.1016/B978-088415317-7/50004-X>

New Mexico Tech. New Mexico Tech Derivative algorithms. Available at <http://infohost.nmt.edu/petro/faculty/Kelly/450/deriv.pdf>. Accessed on April, 15th, 2019.

Nick, H. M.; Raoof, A.; Centler, F.; Thullner, M.; Regnier, P. Reactive dispersive contaminant transport in coastal aquifers: numerical simulation of a reactive Henry problem. **Journal of Contaminant Hydrology**, v. 145, p. 90-104, 2013. <https://doi.org/10.1016/j.jconhyd.2012.12.005>

Offshore Energy. Offshore Energy Today. Available at <https://www.offshoreenergytoday.com/schlumberger-launches-new-well-test-software/>. Accessed on April, 25th, 2019.

Rosa, A. J.; Carvalho, R. S.; Xavier, J. A. D. **Engenharia de Reservatórios de Petróleo**. Rio de Janeiro, Brasil: Interciência, 2006.

Saad, Y. **Iterative Methods for Sparse Linear Systems**. Society of Industrial and Applied Mathematics, USA, 2nd Edition, 2003. <https://doi.org/10.1137/1.9780898718003>

Sanjari, E.; Lay, E. N. An accurate empirical correlation for predicting natural gas compressibility factors. **Journal of Natural Gas Chemistry**, v. 21, p. 184-188, 2011. [https://doi.org/10.1016/S1003-9953\(11\)60352-6](https://doi.org/10.1016/S1003-9953(11)60352-6)

Sutton, R. Compressibility factors for high-molecular-weight reservoir gases. **SPE Annual Technical Meeting and Exhibition**, Las Vegas, USA, 22-25 September 1985. <https://doi.org/10.2118/14265-MS>

Tao, H.; Zhang, L.; Liu, Q.; Deng, Q.; Luo, M.; Zhao, Y. An analytical flow model for heterogeneous multi-fractured systems in shale gas reservoirs. **Energies**, v. 11, n. 12, p. 3422, 2018. <https://doi.org/10.3390/en11123422>

Tavares, C. **Combined effect of non-Darcy flow and formation damage on gas well performance in dual-porosity/dual-permeability reservoirs**. Master's thesis, Colorado School of Mines, Golden, 2003.

Thomas, L.; Dixon, T.; Pierson, R. Fractured reservoir simulation. **Society of Petroleum Engineers Journal**, v. 23, n.1, p. 42-54, 1983. <https://doi.org/10.2118/9305-PA>

Tiab, D.; Donaldson, E. C. **Petrophysics, Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties**. Gulf Professional Publishing, Burlington, USA, 2nd Edition, 2004.

Wang, C. **Pressure transient analysis of fractured wells in shale reservoirs**. Master's thesis, Colorado School of Mines, Colorado, USA, 2013.

Wang, L.; Tian, Y.; Wang, C.; Yao, B.; Wang, S.; Winterfeld, P. H.; Wang, X.; Yang, Z.; Wang, Y.; Cui, J.; Wu, S. Advances in improved/enhanced oil recovery technologies for tight and shale reservoirs. **Fuel**, v. 210, p. 425-445, 2017. <https://doi.org/10.1016/j.fuel.2017.08.095>

Warren, J.; Root, P. The behavior of naturally fractured reservoirs. **Society of Petroleum Engineers Journal**, v. 3, n. 3, p. 245-255, 1963. <https://doi.org/10.2118/426-PA>